Published by IOP Publishing for SISSA
Received: *September 3, 2008*
Accepted: *November 5, 2008*
Published: *November 21, 2008*

Top quark induced vacuum misalignment in little higgs models

Benjamín Grinstein and Michael Trott

University of California, San Diego, Dept. of Physics,
La Jolla, CA 92093-0315, U.S.A.
E-mail: bgrinstein@ucsd.edu, mrtrott@physics.ucsd.edu

Abstract: We show that the effect of the top quark can dominate over the effect of the gauge sector in determining the vacuum alignment in little higgs (LH) models. We demonstrate that in the littlest LH model and the $\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ LH model, ensuring that the correct vacuum alignment is chosen requires that a subset of the gauge sector couplings be large to overcome the effect of the top quark. We quantify this effect by deriving bounds on the couplings in the gauge sector and demonstrate that these bounds provide a compelling theoretical reason for the gauge coupling constant hierarchy in the $\mathrm{SU}(2) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ model that reduces the Goldstone decay constant scale to a TeV. We also argue that for a class of LH models with T parity the top quark drives the correct vacuum alignment and therefore all gauge couplings can be small.

Keywords: Higgs Physics, Beyond Standard Model, Phenomenological Models, Technicolor and Composite Models.

© SISSA 2008

JHEP11(2008)064

Contents

1. Introduction

Little Higgs (LH) models offer an alternative to the standard model in which no fundamental scalars need be introduced (for reviews see [1]). Generally, in LH models the Higgs is a composite particle, bound by interactions that become strong at a scale Λ. The mass of the Higgs is much less than Λ as the Higgs is a pseudo-Goldstone boson (PGB) of broken global symmetries in the theory of the new strong interaction.

The global "flavor" symmetry G_f of these models has a subgroup G_w that is weakly gauged. In the absence of this weak gauge force, the flavor symmetry is broken spontaneously to a subgroup H due to the strong interactions at the scale Λ. As a result, there are massless Goldstone bosons that are coordinates on the G_f/H coset space. Including the effect of G_w as a perturbation, a particular vacuum can be selected in G_f/H which has a particular calculable spectrum of Goldstone bosons. Determining the vacuum selected due to the G_w perturbation is the "Vacuum Alignment" problem [2].

Whether part or all of G_w is spontaneously broken depends on the vacuum alignment. Once the vacuum degeneracy is lifted by the G_w perturbation, there are no more massless Goldstone Bosons; instead there are would-be Goldstone bosons that are eaten by the broken generators of G_w and PGB's whose masses vanish with the couplings in G_w. The Higgs is the lightest PGB in LH models, and its mass is naturally much less than Λ due to the collective symmetry breaking mechanism.

Realistic models identify G_w with the electroweak interactions, or with a larger group that contains the electroweak interactions. They also include interactions responsible for quark masses. It is customary to address the vacuum alignment problem in LH models by first analysing the effect of G_w and then verifying that the effect of other interactions, like those giving quarks their masses, do not destabilize the solution.

JHEP11(2008)064

However the top quark Yukawa coupling is larger than the electroweak gauge couplings. The top quark effect on vacuum alignment can be larger than that of the weakly gauged interactions, even dominant. If in the absence of G_w the selected vacuum is different from the one chosen by G_w alone, and if this different vacuum leads to the wrong low energy spectrum, then the only way to insure the vacuum that gives the SM at low energies is selected is to make the gauge couplings in G_w large enough that they dominate the top quark effect.

In this paper we explore this observation explicitly in the littlest higgs [3] model ($\mathrm{L}^2\mathrm{H}$) and two of its variants. In the $\mathrm{L}^2\mathrm{H}$ we will show that to lowest order in the top Yukawa there are two inequivalent degenerate vacua, a "good" vacuum alignment ($\Sigma_{\rm ew}$) that contains the SM electroweak theory in its low energy limit and a "bad" vacuum alignment (Σ_B) that does not. We then show that to next order in the top Yukawa, the "bad" vacuum is favoured. Including the effect of gauge interactions, we derive a bound on the gauge couplings that must be satisfied if the model is to have $\Sigma_{\rm ew}$ as a global minimum. Having established the need to consider the effects of the top on vacuum alignment in the $\mathrm{L}^2\mathrm{H}$ model, we then consider the effect of the top on vacuum misalignment in variants of the $\mathrm{L}^2\mathrm{H}$ model that are more phenomenologically viable. We derive another bound for the $\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ LH model [4] and note that the bound supplies a reason for the gauge coupling hierarchy $g_1 \gg g_2$ that is desirable as it minimizes the fine tuning of the Higgs mass. We also argue that for LH models with T-parity, the $\Sigma_{\rm ew}$ vacuum is the absolute minimum at lowest order in the top-Yukawa, demonstrating that for some LH model variants, the vacuum selected by the gauge sector can be valid for small gauge couplings.

2. Top vacuum misalignment for $\mathrm{L}^2\mathrm{H}$

To establish notation we briefly review elements of the $\mathrm{L}^2\mathrm{H}$ [3]. It has $G_f = \mathrm{SU}(5)$, $H = \mathrm{SO}(5)$ and $G_w = \prod_{i=1,2}\mathrm{SU}(2)_i \times \mathrm{U}(1)_i$. Symmetry breaking $\mathrm{SU}(5) \to \mathrm{SO}(5)$ is characterized by the Goldstone boson decay constant F. The embedding of G_w in G_f is fixed by taking the generators of $\mathrm{SU}(2)_1$ and $\mathrm{SU}(2)_2$ to be

$$Q_1^a = \begin{pmatrix} \frac{1}{2}\tau^a & 0_{2\times 3} \\ 0_{3\times 2} & 0_{3\times 3} \end{pmatrix} \qquad \text{and} \qquad Q_2^a = \begin{pmatrix} 0_{3\times 3} & 0_{3\times 2} \\ 0_{2\times 3} & -\frac{1}{2}\tau^{a*} \end{pmatrix} \tag{2.1}$$

and the generators of the $\mathrm{U}(1)_1$ and $\mathrm{U}(1)_2$

$$Y_1 = \frac{1}{10}\mathrm{diag}(-3,-3,2,2,2) \qquad \text{and} \qquad Y_2 = \frac{1}{10}\mathrm{diag}(-2,-2,-2,3,3). \tag{2.2}$$

The vacuum manifold is characterized by a unitary, symmetric 5×5 matrix Σ. We denote by g_i (g_i') the gauge couplings associated with $\mathrm{SU}(2)_i$ ($\mathrm{U}(1)_i$). If one sets $g_1 = g_1' = 0$ the model has an exact global SU(3) symmetry (acting on upper 3×3 block of Σ), while for $g_2 = g_2' = 0$ it has a different exact global SU(3) symmetry (acting on the lower 3×3 block). This gives rise to the collective symmetry that ensures the absence of 1-loop quadratic divergences in the higgs mass. To lowest order in the G_w couplings, the quadratically divergent contribution to the vacuum energy is

$$V_w(\Sigma) = \frac{3}{4}\,c\,F^4 \sum_\alpha g_\alpha^2 \mathrm{Tr}\left(T^\alpha \Sigma (T^\alpha)^T \Sigma^\dagger\right), \tag{2.3}$$

JHEP11(2008)064

where the sum on α runs over all generators of G_w. We have normalized so that $c = 1$ corresponds to the quadratic divergence in the Coleman-Weinberg potential with a Euclidean momentum cut-off $\Lambda = 4\pi F$.

It is standard to introduce the top quark so that the collective symmetry argument still applies. Additional spinor fields are introduced: q_R, u_L and u_R that transform as $\mathbf{1}_{2/3}$ under $\mathrm{SU}(2)_1 \times \mathrm{U}(1)_1$, and q_L transforming as $\mathbf{2}_{1/6}$. These couple via

$$\mathcal{L}_{\mathrm{top}} = -\frac{1}{2}\,\lambda_1\, F\, \bar{\chi}_{Li}\, \epsilon_{ijk}\, \epsilon_{mn}\, \Sigma_{jm}\, \Sigma_{kn}\; q_R - \lambda_2\, F\, \bar{u}_L\, u_R + \mathrm{h.c.} \tag{2.4}$$

where the indexes i, j, k run over 1,2,3, the indexes m, n over $4, 5$ and the triplet χ_L is

$$\chi_L = \begin{pmatrix} -i\tau^2 q_L \\ u_L \end{pmatrix}. \tag{2.5}$$

The vacuum energy is determined by the Coleman-Weinberg potential. Using a momentum cut-off in Euclidean space, $|p_E| \leq \Lambda$, it is given by

$$V_t(\Sigma) = -\frac{N_c}{16\pi^2}\left[2\Lambda^2 \mathrm{Tr} M^\dagger M + \mathrm{Tr}(MM^\dagger)^2 \ln(MM^\dagger/\Lambda^2) - \frac{1}{2}\mathrm{Tr}(MM^\dagger)^2\right], \tag{2.6}$$

where $M = M(\Sigma)$ is the spinor mass matrix from eq. (2.4) and $N_c = 3$ is the number of colors. The quadratic and logarithmic divergences are cut-off by modes of the UV completion of the model. Even if we specified the UV completion we would be unable to compute the precise cut-offs, so we parametrize them using $\Lambda = 4\pi F$ and two unknown constants:

$$V_t(\Sigma) = -\frac{N_c}{16\pi^2}\left[2c'\Lambda^2 \mathrm{Tr} M^\dagger M + \mathrm{Tr}(MM^\dagger)^2 \ln(MM^\dagger/\Lambda^2) - \frac{1}{2}\hat{c}'\mathrm{Tr}(MM^\dagger)^2\right]. \tag{2.7}$$

The vacuum energy V_t has two vacua[1] that are degenerate at leading order. The vacuum alignment that leads to $\prod_{i=1,2} \mathrm{SU}(2)_i \times \mathrm{U}(1)_i \to \mathrm{SU}(2) \times \mathrm{U}(1)$ is

$$\Sigma_{\mathrm{ew}} = \begin{pmatrix} 0 & 0 & \mathbf{1}_{2\times 2} \\ 0 & 1 & 0 \\ \mathbf{1}_{2\times 2} & 0 & 0 \end{pmatrix}. \tag{2.8}$$

The second vacuum alignment that is degenerate with Σ_{ew} at leading order in V_t is

$$\Sigma_B = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{2.9}$$

[1] Considering G_w global transformations on the vacuum matrix representations one can show that these are the only two physically distinct degenerate vacua.

JHEP11(2008)064

The Σ_B vacuum alignment leads to $\prod_{i=1,2} \mathrm{SU}(2)_i \times \mathrm{U}(1)_i \to \mathrm{U}(1) \times \mathrm{U}(1)$. The difference in the vacuum energy between $\Sigma_{\rm ew}$ and Σ_B is

$$V_t(\Sigma_{\rm ew}) - V_t(\Sigma_B) = -\frac{N_c}{16\pi^2}\left[(\lambda_1^2+\lambda_2^2)^2 F^4 \left(\ln\left(\frac{(\lambda_1^2+\lambda_2^2)F^2}{\Lambda^2}\right) - \frac{\hat{c}'}{2}\right) - \left(\lambda_1^4 F^4 \ln\left(\frac{\lambda_1^2 F^2}{\Lambda^2}\right) + \lambda_2^4 F^4 \ln\left(\frac{\lambda_2^2 F^2}{\Lambda^2}\right) - \frac{\hat{c}'}{2}(\lambda_1^4+\lambda_2^4)F^4\right)\right]. \quad (2.10)$$

That this is positive is most easily seen by considering $\Lambda \gg \lambda_{1,2} F$:

$$V_t(\Sigma_{\rm ew}) - V_t(\Sigma_B) \approx \frac{N_c}{8\pi^2}\,\lambda_1^2\,\lambda_2^2\,F^4\,\left[\ln\left(\frac{\Lambda^2}{(\lambda_1^2+\lambda_2^2)F^2}\right) + \frac{\hat{c}'}{2}\right] > 0. \quad (2.11)$$

The difference could be negative, and the $\Sigma_{\rm ew}$ vacuum deeper, if $\hat{c}'$ were large and negative. However, $\hat{c}'$ is expected to be positive, since it corresponds to a shift in the cut-off.

The $\Sigma_{\rm ew}$ vacuum can be restored through the effects of the weak gauge interactions. The vacuum energy from (2.3) gives an additional contribution to the energy difference

$$V_w(\Sigma_{\rm ew}) - V_w(\Sigma_B) = -\frac{3}{16}\,c\,F^4\,\left[g_1'^2 + g_1^2\right]. \quad (2.12)$$

Combining results we obtain the condition for the $\Sigma_{\rm ew}$ vacuum alignment to be deeper than the Σ_B^2 alignment is (for $\Lambda \gg \lambda_{1,2} F$):

$$g_1'^2 + g_1^2 > \frac{2N_c}{3\pi^2 c}\,\lambda_1^2\,\lambda_2^2\,\left[\ln\left(\frac{\Lambda^2}{(\lambda_1^2+\lambda_2^2)F^2}\right) + \frac{\hat{c}'}{2}\right]. \quad (2.13)$$

Note that the vacuum alignment bound only restricts the gauge couplings g_1', g_1, as both of the vacua $\Sigma_{\rm ew}$ and Σ_B have $\mathbf{0}_{2\times 2}$ in the the lower right block of the vacuum alignment matrix. Thus calculations of higher order corrections to the bound will still only restrict these two couplings. We will consider some physical implications of this fact in the following two sections.

2.1 Phenomenological implications of top vacuum misalignment in L^{2}H

The L^{2}H is phenomenologically disfavoured [5] by electroweak precision data (EWPD) but is an excellent toy model to examine some consequences of this new constraint for LH model building. In the $\Sigma_{\rm ew}$ vacuum alignment the top quark Yukawa is given by

$$\lambda_t = \frac{\sqrt{2}\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}} + \mathcal{O}\left(\frac{v^2}{F^2}\right). \quad (2.14)$$

Using $\lambda_t = \sqrt{2}\,m_t/v$, along with $\Lambda = 4\,\pi F$ the bound is

$$g_1'^2 + g_1^2 > \frac{2N_c}{3\pi^2 c}\,\lambda_1^2\,\lambda_2^2\,\left[2\,\ln\left(\frac{4\,\pi\,m_t}{\lambda_1\,\lambda_2\,v}\right) + \frac{\hat{c}'}{2}\right]. \quad (2.15)$$

Minimizing (2.15) is accomplished by minimizing $\lambda_1\,\lambda_2$. Using (2.14) we find the bounds on the proto-Yukawa couplings $\lambda_i \geq (m_t/v)$ (with the lower bound reached as $\lambda_j \to \infty$)

JHEP11(2008)064

or $\lambda_1\lambda_2 \geq 2(m_t/v)^2$. Setting $c = \hat{c}' = 1$ to numerically estimate the strength of the bound we obtain $g_1'^2 + g_1^2 > 0.99$. The bound is a constraint on the couplings at the Goldstone decay constant scale F of the L^{2}H theory. The constraint at the scale F restricts the gauge coupling parameter space available for the lower scale matching.[2] At approximately the EW scale ($v \approx 246\,\text{GeV}$) the couplings in the L^{2}H model reduce to the SM gauge couplings as

$$g_{\text{SU}(2)} = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}, \quad g_Y = \frac{g_1' g_2'}{\sqrt{g_1'^2 + g_2'^2}}. \tag{2.16}$$

These relations, and the measured values of $g_{\text{SU}(2)}$, g_Y ensure that if $g_i, (g_i') \to 4\pi$ then $g_j, (g_j') \to g_{\text{SU}(2)}, (g_Y)$ for the SM to be obtained in the low energy limit.[3]

Generically the bound will be stronger than $g_1'^2 + g_1^2 > 0.99$ as the couplings λ_1, λ_2 need not take on the values that give the minimal bound (in fact the minimal bound can only be obtained by fine tuning these couplings). As one moves away from the minimal case $\lambda_1\lambda_2 = 2\,(m_t/v)^2$, the bound grows almost *quadratically*, forcing a subset of the gauge couplings to be large. For example, for $\lambda_1\lambda_2 = 4\,(m_t/v)^2$ the bound is given by $g_1'^2 + g_1^2 > 2.8$. In addition the bounds depend on the precise values of $c, \hat{c}'$ and if $c < 1$ then the bounds are stronger.

Consider the following example of the consequences of the top misalignment bound. One of the main limitations of the L^{2}H model stems from the amount of custodial symmetry violation present in the model. The largest effect of custodial symmetry violation has a dependence on the gauge couplings of the form [5]

$$\rho = 1 + \frac{v^2}{F^2}\,\frac{5}{4}\left(\frac{g_1'^2 - g_2'^2}{g_1'^2 + g_2'^2}\right)^2 - \frac{v'^2}{v^2}, \tag{2.17}$$

where v' is the triplet scalar vev.[4] Custodial symmetry is minimized when there is a degeneracy of the gauge couplings of the form $g_1'^2 \sim g_2'^2$. Consider this phenomenologically favored region of parameter space where $g_2' = g_1'(1 + a)$ with $a \ll 1$. Then the bound translates into the following new constraint (using $g_Y \approx 0.35$)

$$\frac{g_1^2}{g_1'^2} \geq 3.1 + 4.1\,a + \mathcal{O}(a^2). \tag{2.18}$$

The top quark induced vacuum alignment bounds do remove some of the L^{2}H parameter space with minimal custodial symmetry violation. In the remaining region of favored parameter space, due to the bound, hierarchies must exist among the L^{2}H gauge couplings for Σ_{ew} to be the vacuum of the model. As the bound grows almost quadratically as one moves away from the minimal value of $\lambda_1\,\lambda_2$, we see that the top misalignment bounds select for a gauge coupling hierarchy of the form $g_1 > g_1' \sim g_2' > g_2$ in this favored region of parameter space.

[2] We will neglect the small effect of running when considering the bounds in what follows.

[3] If the strong coupling limit $g_i, (g_i') \to 4\pi$ is reached the ability to conclude anything in perturbation theory (including the bound) is removed. We are considering the case that the the gauge coupling is not driven into the truly strong coupling regime.

[4] We neglect the weak dependence of v' on the gauge couplings.

JHEP11(2008)064

This is an interesting dynamical mechanism, the minimization of the model's energy in selecting the correct vacuum alignment ($\Sigma_{\rm ew}$) requires that a subset of the gauge couplings are large. This idea of a vacuum alignment induced gauge coupling hierarchy can have significant phenomenological consequences as we discuss in the next section.

The top misalignment bound is of marginal interest in the L^{2}H model alone as this model is already so phenomenologically disfavored by EWPD. Further, the bounds can be satisfied and the SM couplings obtained by choosing the L^{2}H gauge and proto-Yukawa couplings judiciously, although this does increase the amount of tuning in the model. As the L^{2}H model is the template on which many LH models are based, it is of interest to investigate these considerations in more phenomenologically viable LH models.

3. Top vacuum misalignment for SU(2) × SU(2) × U(1)

The SU(2)×SU(2)×U(1) LH model is an interesting LH variant. The constraints of EWPD are significantly relaxed in this model compared to the L^{2}H model [5]. By only gauging a single U(1), the heavy U(1) gauge boson of the L^{2}H model is eliminated. This improves the viability of the model as the heavy U(1) gauge boson leads to the $\mathcal{O}(v^2/F^2)$ custodial symmetry violation, and is in fact not very heavy. Its nonobservation at the Tevatron [5] requires a large Goldstone decay constant scale for the L^{2}H model and more fine tuning in the Higgs mass. By only gauging one U(1), the agreement of the model with EWPD is improved at the cost of not removing all one loop quadratic divergences in the Higgs mass. However the remaining quadratic divergence due to the U(1) charge does not lead to significant fine tuning in the Higgs mass for a cut off of $\Lambda \sim 10\,\text{TeV}$. In particular, this residual amount of tuning is reduced for particular choice of the gauge couplings ($g_1 \gg g_2$) in the model. Let us examine this choice of couplings in the light of the top misalignment bound.

The SU(2) × SU(2) × U(1) LH model is substantially the same as L^{2}H except that the generator of the gauged U(1) is given by

$$Y_1 = \frac{1}{2}\text{diag}(1, 1, 0, -1, -1) \tag{3.1}$$

As $\mathcal{L}_{\rm top}$ is identical for the SU(2) × SU(2) × U(1) model, Equations (2.4)–(2.11) are unchanged. The gauge boson mass spectrum is however different and the corresponding contribution to the difference of the vacuum energies is

$$V_w(\Sigma_{\rm ew}) - V_w(\Sigma_B) = -\frac{3}{16} c\, F^4 \left[3\, g_Y^2 + g_1^2\right] \tag{3.2}$$

leading to the bound (for $\Lambda \gg \lambda_{1,2} F$):

$$3\, g_Y^2 + g_1^2 > \frac{2\, N_c}{3\pi^2 c}\, \lambda_1^2\, \lambda_2^2 \left[2\, \ln\left(\frac{4\,\pi\, m_t}{\lambda_1\, \lambda_2\, v}\right) + \frac{\hat{c}'}{2}\right] \tag{3.3}$$

F as low as a TeV is consistent with EWPD in this model [5]. Such a low F (and a correspondingly low Λ) requires $g_1 \gg g_2$ which was theoretically unsatisfying as this hierarchy of gauge couplings had no explanation in the model. The top vacuum misalignment bound supplies a condition that translates into this hierarchy of gauge couplings.

JHEP11(2008)064

Consider the alignment bound and let $\lambda_1\lambda_2 = 2\,x\,(m_t/v)^2$ where $x \geq 1$. The bound on the g_1 coupling is

$$g_1^2 > 0.99\,x^2\,(1 - 0.4\,\log x) - 3\,g_Y^2. \tag{3.4}$$

No constraint requires that g_2 must be as large as g_1. Further, to reduce to the SM's measured value of $g_{\mathrm{SU}(2)}$, as $g_1 \to 4\,\pi$ one must have $g_2 \to g_{\mathrm{SU}(2)}$. Thus, minimizing the energy of the system (with the constraint that the SM is obtained) and reducing the tuning on the proto-Yukawa couplings drives the hierarchy in the gauge coupling $g_1 \gg g_2$. This in turn drives the allowed F down to a TeV in this model, which in turn reduces the remaining tuning required for the Higgs mass!

Generic signals of LH models at LHC include the observation of the heavy top partner and the new heavy gauge bosons and have been extensively studied in the literature [5]. Other LH models can have very similar phenomenology (accessible at LHC) to the SU(2) × SU(2) × U(1) model. It has also been noted that to discriminate between LH models at LHC and to distinguish them from supersymmetry and extra dimension scenarios can be challenging, although strategies exist in the literature [6].

We note that when the scale $F \sim$ TeV in this model, one expects the properties of the Higgs to deviate from its properties in the SM significantly. This is another source of experimental information to aid in the discrimination between models. Integrating out all of the details of this LH model to study physics around the EW scale, one obtains dimension six operators that are suppressed by v^2/F^2 modifying the properties of the pseudo Goldstone Higgs. The operators that are induced at tree level can have interesting phenomenological effects,[5] such as enhancing the $g\,g \to h\,h$ signal at LHC allowing one to measure the Higgs self coupling [7]. Operators of this form can also significantly effect the decay width of the low mass Higgs [8] ($m_h < 140\,\mathrm{GeV}$) suppressing or enhancing the low mass Higgs discovery signal $gg \to h \to \gamma\,\gamma$. Other dimension six operators that this model will induce (suppressed by loop factors) can also effect the production mechanism of the low mass Higgs at LHC and ILC [9]. The Wilson coefficients of all of these operators are restricted by the constraint that the top misalignment bound places on the couplings of the model.

4. Top vacuum alignment bounds and T parity

Another approach to improving the viability of LH constructions is to impose T parity [11] under which all the new heavy gauge bosons and the scalar triplet are odd. This forbids the tree level contributions of these new states to EWPD observables.

In order to study the effect of the top quark on vacuum alignment in LH models with T-parity, a transformation law for the vacuum orientation matrix Σ is needed. Rather than attempting a general argument we content ourselves with a specific example. We consider a model with the same global and gauge symmetries as the L^{2}H. To elucidate the transformation properties of Σ under T-parity we consider a UV completion consisting of a

[5]Such low energy effects on Higgs phenomenology can also be the source of the baryon-antibaryon asymmetry of the universe at the modified electroweak phase transition in the Pseudo-Goldstone Baryogenesis scenario [10].

JHEP11(2008)064

theory with five spinors in a real representation R of a techni-strong gauge interaction such that the symmetric part of $R \times R$ contains a singlet. Collect the spinors into two doublets, $\psi_\pm$, with ψ_+ (ψ_-) a doublet of $\mathrm{SU}(2)_1$ ($\mathrm{SU}(2)_2$), and a singlet ψ_0. Since the action of T parity exchanges $\mathrm{SU}(2)_1 \times \mathrm{U}(1)_1$ with $\mathrm{SU}(2)_2 \times \mathrm{U}(1)_2$, the spinors transform, up to trivial unitary redefinitions, by

$$\psi_\pm \to \psi^c_\mp, \qquad \psi_0 \to \psi^c_0, \tag{4.1}$$

where the superscript "c" denotes charge conjugation. This allows a complete characterization of the transformation of the condensate $\langle\psi\psi\rangle$, but for our purposes it is only necessary to note that if the upper left and lower right 2×2 blocks of Σ are denoted by $A_\pm$, then under T-parity $A_\pm \to A^\dagger_\mp$.

The leading (quadratically divergent) term in the top-induced vacuum energy in (2.7) depends on Σ through a positive definite function of $A^\dagger_- A_-$. Both $\Sigma_{\rm ew}$ and Σ_B alignments in the L^{2}H case are obtained precisely by setting $A_- = 0$. When the top-quark sector is extended to insist on T-parity symmetry, the resulting vacuum energy is symmetric under the exchange $A_\pm \to A_\mp$. The vacuum energy is a sum of two positive definite functions one of $A^\dagger_+ A_+$ and one of $A^\dagger_- A_-$. The minimum energy is obtained by setting $A_+ = A_- = 0$. The vacuum alignment with this property is up to gauge rotations the $\Sigma_{\rm ew}$ vacuum alignment.

There is no top induced misalignment regardless of the strength of the interactions in G_w. In fact, one could consider models in which vacuum alignment is completely driven by the top sector. A curious example has G_w replaced by a single $\mathrm{SU}(2) \times \mathrm{U}(1)$. The collective symmetry argument applies exactly in the gauge sector so the higgs potential arises only from the top-quark sector induced Coleman-Weinberg potential. However, in this model $\mathrm{SU}(2) \times \mathrm{U}(1)$ is completely broken at the scale F. Alternatively one can use for the gauged group the diagonal sum of the generators of the L^{2}H model. This remains unbroken, but has no collective symmetry to prevent radiative contributions to the higgs mass quadratic in F.

5. Cosmological considerations

We have derived our bound insisting on absolute stability of the $\Sigma_{\rm ew}$ vacuum as it is likely that as the universe cools it picks the stable vacuum. In a cosmological setting one may relax this condition by considering metastability [12].

Consider the case that the bound (2.13) is violated and the $\Sigma_{\rm ew}$ vacuum is selected as the universe cools down in the L^{2}H model. If this occurs the $\Sigma_{\rm ew}$ vacuum solution is not the absolute minimum vacuum solution and is metastable. A reliable computation of the lifetime of the metastable vacuum requires understanding the shape of the potential, which depends on many field variables. This is beyond the scope of this work. A rather crude estimate is obtained as follows. We take the height of the potential barrier to be F^4 and the distance on field space between the vacua to be F. Then the condition that the metastable vacuum does not decay within the age of the universe, t_0, is

$$\frac{(t_0 F)^4}{4\pi^2} \left(\frac{k}{\epsilon^3}\right)^2 e^{-k/\epsilon^3} \lesssim 1 \tag{5.1}$$

JHEP11(2008)064

where $k = (4\pi)^2(16/3)^3$ and ϵ is the difference between the right hand side and the left hand side of (2.13). This condition is satisfied for a wide range of couplings that violate our bound. However, for our bound to be violated this way requires the $\Sigma_{\rm ew}$ vacuum to be selected over the Σ_B vacuum as the universe cools down. Determining whether this is the case requires knowledge of the UV completion and thermal evolution of the theory.

By the same token, consider the case that the bound (2.13) is satisfied and yet the universe cools down into the Σ_B vacuum. Then the Σ_B vacuum solution would be metastable. The lifetime of this metastable vacuum is determined just as above and hence can easily be longer than the age of the universe. This possibility is largely insensitive to the particular parameters of the theory.

To the extent that the crude estimate of the lifetime of the metastable vacua is reliable, we conclude that it is the thermal evolution that determines the vacuum of the universe when metastability is considered.

6. Conclusions

We have shown that the effect of the top quark on vacuum alignment can dominate the effect of the gauge interactions in selecting the vacuum alignment of LH theories. We have demonstrated that to ensure that the low energy limit of the LH model reduces to the standard model, one can derive bounds on the size of particular LH model's gauge couplings. We have derived such bounds for the $\mathrm{L}^2\mathrm{H}$ and $\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ LH models.

In examining the consequences of these bounds, we have observed that they require that a subset of gauge couplings of the LH model must be large. Such hierarchies in gauge couplings can have significant phenomenological consequences; the hierarchy $g_1 \gg g_2$ in the $\mathrm{SU}(2)\times\mathrm{SU}(2)\times\mathrm{U}(1)$ LH model that is phenomenologically appealing can be justified by the top induced vacuum misalignment bound.

By considering a particular UV completion on LH models with T parity, we have argued that the top misalignment bound does not constrain the gauge couplings of the model.

This work can be extended in many ways. The effect of top induced vacuum misalignment on many other LH models should be examined. Further, one can examine the bounds at higher order in the gauge and proto-Yukawa couplings to further refine the bounds and the effect of running between the scale F and the scale v can be incorporated. It would also be interesting to use the top sector to drive the vacuum alignment in model building where T parity is imposed and to investigate further the possibility of metastability.

Acknowledgments

Work supported in part by the US Department of Energy under contract DE-FG03-97ER40546.

References

[1] M. Schmaltz and D. Tucker-Smith, *Little Higgs review*, *Ann. Rev. Nucl. Part. Sci.* **55** (2005) 229 [hep-ph/0502182];

JHEP11(2008)064

M. Perelstein, *Little Higgs models and their phenomenology*, *Prog. Part. Nucl. Phys.* **58** (2007) 247 [hep-ph/0512128];
H. Georgi, *The Higgs as a pseudo-Goldstone boson*, *Comptes Rendus Physique* **8** (2007) 1029.

[2] J. Preskill, *Subgroup alignment in hypercolor theories*, *Nucl. Phys.* **B 177** (1981) 21;
M.E. Peskin, *The alignment of the vacuum in theories of technicolor*, *Nucl. Phys.* **B 175** (1980) 197.

[3] N. Arkani-Hamed, A.G. Cohen, E. Katz and A.E. Nelson, *The littlest Higgs*, *JHEP* **07** (2002) 034 [hep-ph/0206021].

[4] M. Perelstein, M.E. Peskin and A. Pierce, *Top quarks and electroweak symmetry breaking in little Higgs models*, *Phys. Rev.* **D 69** (2004) 075002 [hep-ph/0310039].

[5] T. Han, H.E. Logan, B. McElrath and L.-T. Wang, *Phenomenology of the little Higgs model*, *Phys. Rev.* **D 67** (2003) 095004 [hep-ph/0301040];
C. Csáki, J. Hubisz, G.D. Kribs, P. Meade and J. Terning, *Big corrections from a little Higgs*, *Phys. Rev.* **D 67** (2003) 115002 [hep-ph/0211124];
J.L. Hewett, F.J. Petriello and T.G. Rizzo, *Constraining the littlest Higgs*, *JHEP* **10** (2003) 062 [hep-ph/0211218];
C. Csáki, J. Hubisz, G.D. Kribs, P. Meade and J. Terning, *Variations of little Higgs models and their electroweak constraints*, *Phys. Rev.* **D 68** (2003) 035009 [hep-ph/0303236];
R. Barbieri, A. Pomarol, R. Rattazzi and A. Strumia, *Electroweak symmetry breaking after LEP1 and LEP2*, *Nucl. Phys.* **B 703** (2004) 127 [hep-ph/0405040].

[6] A. Belyaev et al., *Dictionary of LHC signatures*, arXiv:0806.2838;
K. Hsieh and C.P. Yuan, *Lone Higgs at the LHC*, *Phys. Rev.* **D 78** (2008) 053006 [arXiv:0806.2608];
P. Meade and M. Reece, *Top partners at the LHC: spin and mass measurement*, *Phys. Rev.* **D 74** (2006) 015010 [hep-ph/0601124];
H.-C. Cheng, K.T. Matchev and M. Schmaltz, *Bosonic supersymmetry? Getting fooled at the LHC*, *Phys. Rev.* **D 66** (2002) 056006 [hep-ph/0205314].

[7] B. Grinstein and M. Trott, *A Higgs-Higgs bound state due to new physics at a TeV*, *Phys. Rev.* **D 76** (2007) 073002 [arXiv:0704.1505].

[8] S. Mantry, M. Trott and M.B. Wise, *The Higgs decay width in multi-scalar doublet models*, *Phys. Rev.* **D 77** (2008) 013006 [arXiv:0709.1505];
S. Mantry, M.J. Ramsey-Musolf and M. Trott, *New physics effects in Higgs decay to tau leptons*, *Phys. Lett.* **B 660** (2008) 54 [arXiv:0707.3152];
L. Randall, *Two Higgs models for large* $\tan\beta$ *and heavy second Higgs*, *JHEP* **02** (2008) 084 [arXiv:0711.4360].

[9] G.J. Gounaris, J. Layssac and F.M. Renard, *Testing the Higgs boson gluonic couplings at LHC*, *Phys. Rev.* **D 58** (1998) 075006 [hep-ph/9803422];
A.V. Manohar and M.B. Wise, *Modifications to the properties of a light Higgs boson*, *Phys. Lett.* **B 636** (2006) 107 [hep-ph/0601212];
J. Kile and M.J. Ramsey-Musolf, *Fermionic effective operators and Higgs production at a linear collider*, *Phys. Rev.* **D 76** (2007) 054009 [arXiv:0705.0554].

[10] C. Delaunay, C. Grojean and J.D. Wells, *Dynamics of non-renormalizable electroweak symmetry breaking*, *JHEP* **04** (2008) 029 [arXiv:0711.2511];
B. Grinstein and M. Trott, *Electroweak baryogenesis with a pseudo-Goldstone Higgs*, *Phys. Rev.* **D 78** (2008) 075022 [arXiv:0806.1971].

JHEP11(2008)064

[11] H.-C. Cheng and I. Low, *TeV symmetry and the little hierarchy problem*, *JHEP* **09** (2003) 051 [hep-ph/0308199]; *Little hierarchy, little Higgses and a little symmetry*, *JHEP* **08** (2004) 061 [hep-ph/0405243].

[12] S.R. Coleman, *The fate of the false vacuum.* 1. *Semiclassical theory*, *Phys. Rev.* **D 15** (1977) 2929 [*Erratum ibid.* **D 16** (1977) 1248];
C.G. Callan Jr. and S.R. Coleman, *The fate of the false vacuum.* 2. *First quantum corrections*, *Phys. Rev.* **D 16** (1977) 1762;
S.R. Coleman and F. De Luccia, *Gravitational effects on and of vacuum decay*, *Phys. Rev.* **D 21** (1980) 3305.

JHEP11(2008)064